J. Fluid Mech. (1971), *vol.* 47, *part* 4, *pp.* 765–777
Printed in Great Britain

Static and dynamic calibrations of constant-temperature hot-wire systems

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(Received 24 June 1970)

The conventional hot-wire static calibration procedure for the measurement of absolute turbulence intensities with constant-temperature hot-wire anemometers is investigated and serious errors are found. An alternative calibration procedure is developed which involves shaking the wire at low frequencies in a uniform flow. A series of tests indicate that this dynamic calibration method is more accurate and consistent than the conventional procedure.

A method for verifying various calibration procedures is demonstrated. This method involves the measurement of velocity perturbations in a series of Karman vortex streets. The velocity perturbation amplitude is held fixed, but the frequency varies from one vortex street to another. This method also acts as a direct check of the hot-wire system frequency response.

1. Introduction

The results of this paper stem from an attempt to measure the frequency response of hot-wire systems by a direct velocity perturbation method rather than an electronic method. Electronic techniques for simulating velocity perturbations rely on certain assumptions regarding the behaviour of the electronic circuit and the results can be misleading. Various methods for producing velocity perturbations of fixed amplitude and variable frequency were tried. The most successful method involved taking traverses of velocity fluctuations across Karman vortex streets. It was discovered that during these tests commercial hot-wire systems showed apparently very poor frequency response (3 db down at 1 kHz). The authors then designed and built a hot-wire system which followed the assumed electronic system model closely (see Perry & Morrison 1971). Electronic tests indicated that the frequency response of this system to velocity perturbations was very high. However, when the direct velocity perturbation test was applied to this system the response was apparently poor. It was discovered that erroneous frequency-response results were obtained because the calibration of d.c. response against speed was incorrect, leading to an apparent variation of response with frequency at constant Strouhal number. This method involved the use of the standard plot of the static quantities E_0^2 versus U^n, where E_0 is the mean output voltage, U is the mean velocity and n is an index whose value depends on which heat-transfer law the user may favour. The usual form is given by

$$E_0^2 = A + BU^n, \tag{1}$$

and A and B are constant for a given wire, fluid and wire resistance ratio. The King's law case corresponds to $n = 0{\cdot}5$. The slope of the straight line of best fit to the calibration data on an E_0^2 versus U^n plot enables the sensitivity of the hot-wire system to be determined.

An alternative calibration method is to shake the wire sinusoidally with a known velocity perturbation at low frequency in a uniform stream. When this calibration method was applied to the vortex street tests, the hot-wire systems tested showed improved frequency response. These responses were of the same order as the electronic test results.

Dryden & Kuethe (1929) used a similar shaking method for producing known low-frequency velocity perturbations and compared the results with those obtained using the static calibration procedure. A discrepancy was noted but was dismissed as experimental error.

2. Direct determination of hot-wire system frequency response

The usual method for determining the frequency response of a hot-wire system is to excite the circuit with a square-wave voltage perturbation and observe the transient response. This will give the system poles, from which the roll-off frequency and damping coefficient may be evaluated but no information regarding the system zeros can be obtained. To determine these zeros it is necessary to perform a steady-state test with sinusoidal voltage perturbations. It is also necessary to know the connexion between the two transfer functions involved. One transfer function relates the output voltage to the input velocity perturbation while the other relates the output voltage to the input excitation voltage (Perry & Morrison 1971).

To ensure that there are no zeros or unaccounted phenomena occuring at frequencies below the lowest frequency poles, a more direct test would be desirable. One previous attempt to devise a more direct test than the electronic method was that of Kidron (1966), who simulated velocity perturbations by heating the hot-wire element with modulated electromagnetic energy. Kidron checked the constant-current characteristics of a hot wire and hot film but did not report any tests of the detailed response of a closed-loop hot-wire system. The most direct test is to use a velocity field with a known velocity perturbation and with controllable frequency. This may be achieved by vibrating the hot-wire probe on a high-frequency vibrating table. The response may be checked by comparing the hot-wire signal with the integrated signal from an accelerometer which moves with the probe. This was tried by the authors but was unsuccessful due to unexpected low-frequency peaks (< 1 kHz). These low-frequency resonances are thought to be due to vibration of the hot-wire element relative to the prongs, caused by the inertia forces on the wire. Various aspects of these tests are still under investigation. Similar tests were performed by Bellhouse & Rasmussen (1968) and they experienced the same difficulties with resonances in the hot-wire filament, but were more successful with hot films.

Another technique for generating the required flow conditions is to use the

Karman vortex street shed from a cylinder. The r.m.s. value of the velocity perturbation u in the cylinder wake is given by

$$\frac{(\overline{u^2})^{\frac{1}{2}}}{U_\infty} = \phi_1\left(R_N, \frac{x}{d}, \frac{y}{d}\right), \tag{2}$$

where U_∞ is the free stream velocity, R_N the cylinder Reynolds number, x/d the number of cylinder diameters downstream from the cylinder and y/d the number of cylinder diameters off the wake centre line. The Strouhal number S_N is given by

$$S_N = \frac{fd}{U_\infty} = \phi_2(R_N), \tag{3}$$

where throughout this paper f is taken as the vortex shedding frequency from one side of the cylinder. Thus if a range of cylinder diameters are used, and the Reynolds number is maintained constant, the $(\overline{u^2})^{\frac{1}{2}}/U_\infty$ versus y/d profiles should

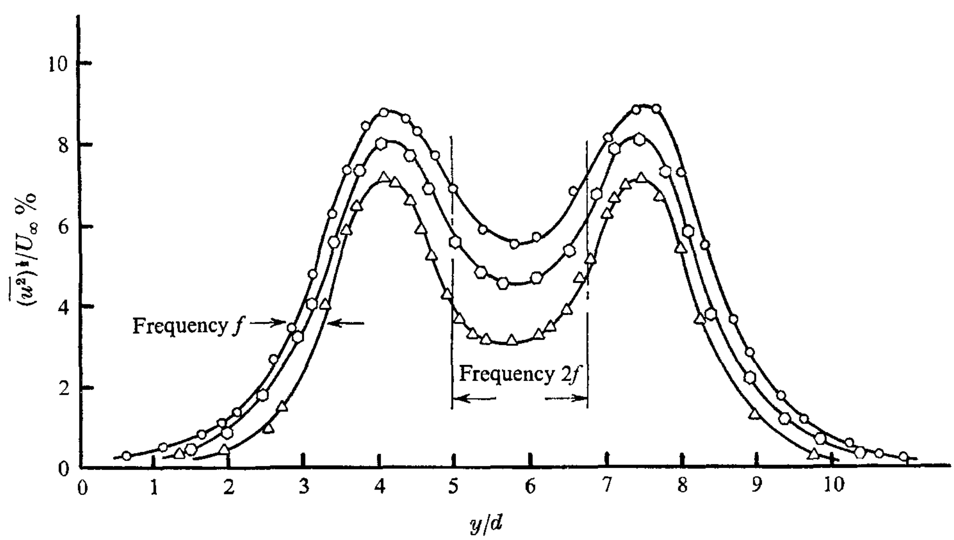

FIGURE 1. Velocity perturbation profiles as measured using Disa 55A01 system. $R_n = 140$. (Static King's law calibration.) Frequency f (Hz): ○, 500; ●, 1500; △, 4000. Origin for y is arbitrary.

be identical at a fixed x/d. This should be true even though (2) and (3) show that the frequency f is proportional to $1/d^2$ for the case of constant Reynolds number. When the conventional King's law and static calibration is used, the results of the cylinder wake measurements are as shown in figure 1 for a Disa type 55A01 constant-temperature system. Similar results were obtained using the constant-temperature system constructed by the authors. Throughout all tests reported here Wollaston-type platinum wire was used. The wire and probe details are shown in figure 6.

There are a number of possible explanations for the deviation of these results from the correlation defined by (2).

(1) The frequency response of both systems to velocity perturbations is less than 1 kHz. This conflicts with the results of the square-wave voltage stimulation

tests which indicate that the roll-off frequencies were 5 kHz for the Disa and 35 kHz for the authors' system.

(2) The calibration procedure introduces an error which is a function of the bias velocity. This would cause the non-dimensional profiles to differ even if the system frequency response is flat.

(3) There are other variables influencing the flow. Cylinder length to diameter ratio, background turbulence and cylinder vibration are possible effects. The filtering effect due to the finite length of the wire could become serious if the vortices were inclined steeply to the wire or if the wire was misaligned. Further investigation showed this to be negligible for the cases studied. The other effects were discounted when the measurements were found to be repeatable over a wide range of cylinder length to diameter ratios and background turbulence levels. Cylinder vibration was discounted by changing the tension in the vortex shedding cylinder and repeating the measurements.

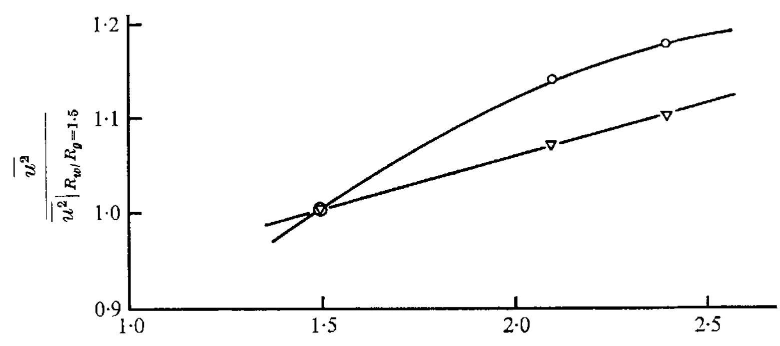

FIGURE 2. Turbulence measurements as a function of resistance ratio using King's law calibration. ○, Disa 55A01; ▽, authors' system.

To check the consistency of the calibration procedure, the system response to a given turbulence field was measured a number of times with one hot wire operated at different wire temperatures and hence different resistance ratios R_w/R_g. Here R_w is the operating wire resistance and R_g is the wire resistance at ambient temperature. The system was calibrated statically using the King's law form of (1) at each of the wire resistances used. The probe was then placed in the wake behind a cylinder and the turbulence level measured and repeated for each of the calibrated wire resistances without shifting the probe or changing the flow conditions. Figure 2 shows the results for the Disa and the authors' systems. It can be seen that the values as determined from the King's law calibration are a function of the wire resistance. Ideally all these results should be identical.

3. Conventional calibration procedure

The conventional method for evaluating the small perturbation calibration constant $\partial E_0/\partial U$ is to plot the calibration data as E_0^2 versus U^n as suggested by the functional form of (1). The data is then graphically differentiated by determining the straight line of best fit. Alternatively, a numerical method could be used for fitting (1) to the data. The constant B in (1) is equal to the slope of this straight line and the calibration constant may be evaluated as follows:

$$\frac{\partial E_0}{\partial U} = \frac{nBU^{n-1}}{2E_0}. \tag{4}$$

This technique requires the operator to know the value of n. The quoted values of n vary from one authority to another but it is believed to be in the range 0·4–0·5. Collis & Williams (1959) suggested that $n = 0{\cdot}45$ for straight hot wires

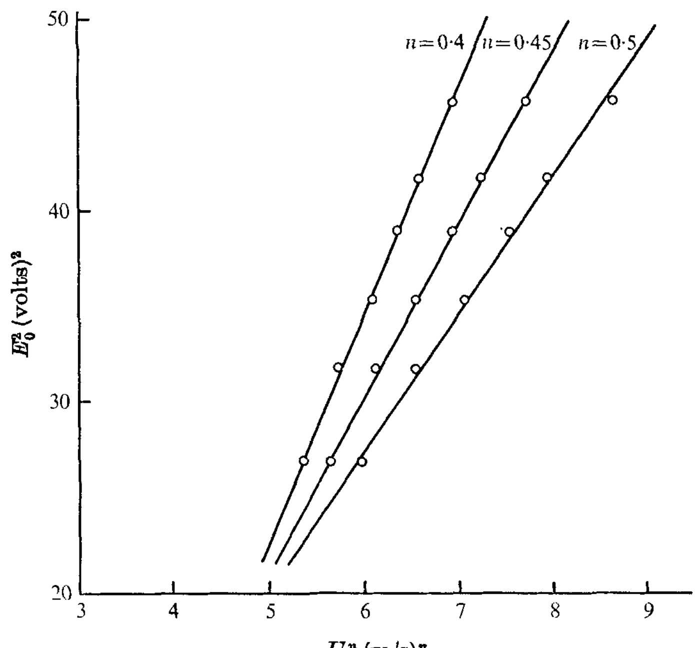

FIGURE 3. Alternative forms of static calibration. $E_0^2 = A + BU^n$.

with large length to diameter ratios ($l/d > 2000$). As a result of their work many operators have used $n = 0{\cdot}45$ rather than the original King's law value of $n = 0{\cdot}5$. From the authors' experience no value of n could be found which satisfactorily correlates the data obtained from a real hot wire (which is usually slightly bowed due to thermal expansion and whose length to diameter ratio is typically 200:1). To demonstrate this, a series of calibration data obtained for one fixed resistance ratio using the Disa 55A01 system is plotted in figure 3, using $n = 0{\cdot}4$, 0·45 and 0·5 alternatively. The absolute value of the deviation of the points from the straight line of best fit is similar for each value of n, but it will be shown later, in § 5, that the small perturbation sensitivity as determined from the gradient of each of the three straight lines may vary with n by up to 20 %. If the $U = 0$ calibration point is included in the curve fit, the variation of the sensitivity will be larger. These results show that the heat-transfer function is not described satisfactorily by (1) since there are a range of lines the operator could take as the straight line of best fit because the calibration depends on the range over which the operator assumes (1) applies.

Some experimenters assume that the gradient of the heat-transfer law may be determined by taking one calibration point at $U = 0$ and another at some high velocity (Bullock & Bremhorst 1969). Others assume that the calibration deviates from a straight line on an E_0^2 versus U^n plot at low velocities and thus the $U = 0$ point may not be used (Rasmussen 1965; Almquist & Legarth 1965).

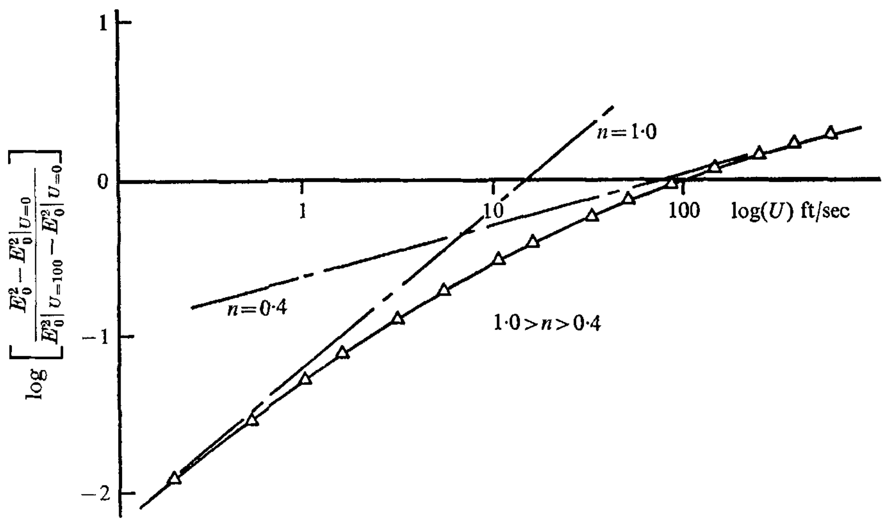

FIGURE 4. Variation of index n with mean velocity. (After Davies 1968.) $E_0^2 = A + BU^n$

Other experimenters have observed that the correlation as given by (1) does not apply at very low or very high Reynolds numbers (Norman 1967), and the limits are not well defined since they depend on other parameters such as Knudsen number (Davis 1968). The range over which the law is applicable has to be estimated from the appearance of the data and thus the method becomes subjective. This was illustrated by presenting three operators with the same calibration data and observing that the calculated sensitivities varied by 15 % between operators.

Davis (1968) demonstrated that a possible explanation for the deviation of the calibration from (1) is that the correlation scheme is unsuitable since n is a function of the mean velocity. This was shown by plotting data on log $(E_0^2 - E_0^2|_{U=0})$ versus log (U) axes and is shown in figure 4. Other possible reasons for the inadequacy of the method is that the heat transfer from a real hot wire will depend on such things as wire bowing, length to diameter ratio, departure of wire cross-sectional shape from a circle, end conduction, wire temperature, and the fluid variables associated with variations of viscosity and density with temperature. If all these effects were taken into account, the heat-transfer law would probably be too complex to be of practical use. To avoid these problems the authors developed an alternative calibration procedure which does not require knowledge of the form of the heat-transfer law, leaves no latitude for operator bias and avoids the need for graphical or numerical differentiation of data which invariably has scatter.

4. Small perturbation calibration

The graphical or numerical differentiation of the static calibration data may be avoided by subjecting the hot-wire element to an accurately known small perturbation ΔU of the bias velocity and observing the change of the hot-wire output voltage ΔE_0. If ΔU is sufficiently small the approximation

$$\partial E_0/\partial U \simeq \Delta E_0/\Delta U$$

may be used. It is difficult to measure ΔE_0 and ΔU statically, due to the large bias values of both U and E_0 but these quantities may be determined simply and accurately if dynamic sinusoidal velocity perturbations are generated and the hot-wire output is a.c. coupled. Accurately known velocity perturbations may be generated by attaching the hot wire to a dynamically balanced shaker, provided the frequency of shaking is well below resonances of the hot-wire element and probe. The authors constructed such a dynamic calibrator, the plan and elevation of which are shown in figure 5. The calibrator gives a peak velocity perturbation in the range of 0·3 to 5 m/s at frequencies of 1–15 Hz respectively. To avoid unwanted vibrations, the system was balanced dynamically by using a pair of scotch yokes with co-axial counter oscillating shafts. While in operation the calibrator is held outside the working section of a low turbulence wind tunnel and an aerofoil section sting transfers the motion from the oscillating shaft to the hot-wire probe in the free stream.

The calibrator instrumentation is shown in figure 6. The r.m.s. value of the hot-wire output voltage is measured by an analog computer which squares and integrates the signal for a known time T. The r.m.s. calibrator speed is measured by using a photocell and slotted flywheel to drive an electronic counter which records the total number of revolutions of the system during the time interval T. By measuring the r.m.s. calibrator speed and the hot-wire r.m.s. output voltage during the same time lapse T, the sensitivity may be evaluated for the given velocity and stream temperature. To ensure linear operation of the hot wire, the calibrator speed is always adjusted to be less than 10 % of the bias velocity. The effects of large amplitude calibrating velocities are discussed in § 6. The variation of the system sensitivity with mean velocity may be determined by repeating the calibration over a range of bias speeds.

An important requirement of this technique is that the low-frequency roll-off of the r.m.s. measuring system due to a.c. coupling should be well below the lowest calibrator frequency and that the hot-wire system itself should have a flat response to low frequencies. This was checked by calibrating the system over the full frequency range of the calibrator with the wire exposed to a high bias speed U such that at its highest frequency the peak velocity perturbation was less than $0{\cdot}1U$. Both the Disa 55A01 and the authors' system showed a consistency of sensitivity to within $\frac{1}{4}$ % over the full frequency range of the calibrator.

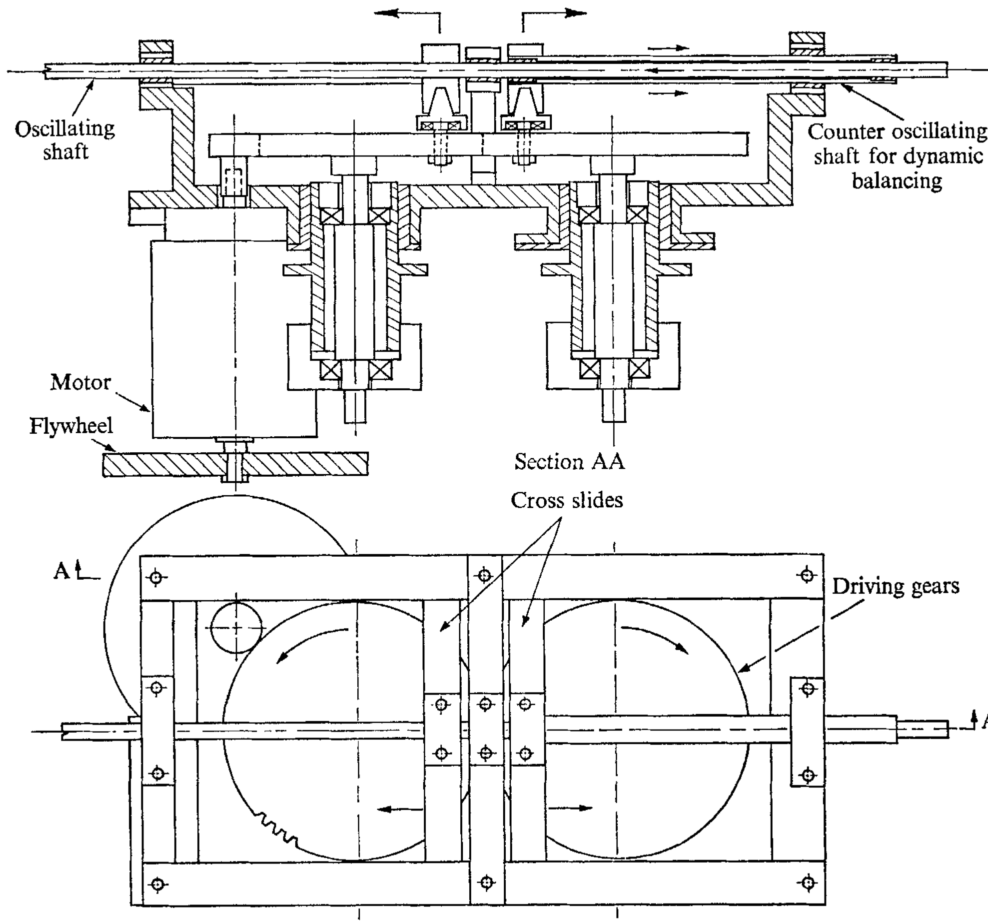

FIGURE 5. Dynamic calibrator.

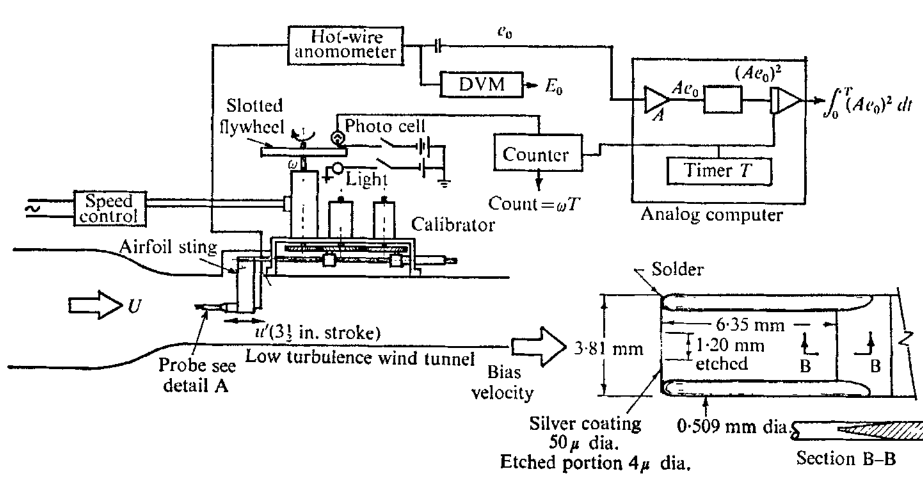

FIGURE 6. Dynamic calibrator instrumentation and probe details. Wire material used was platinum (Wollaston type).

5. Comparison of conventional and small perturbation calibrations

The system sensitivity $\partial E_0/\partial U$ for a typical platinum hot wire $4\,\mu$ in diameter and 1·2 mm long as determined statically using equations (1) and (4) with $n = 0{\cdot}4$, 0·45 and 0·5 and from the dynamic calibrator are compared in figure 7. These results demonstrate a substantial difference between the conventional static method and the dynamic calibration method. The difference is a function of the bias velocity and may be larger than 20 % at high speeds. It can also be seen that the results of the static calibration are very sensitive to changes of the index n.

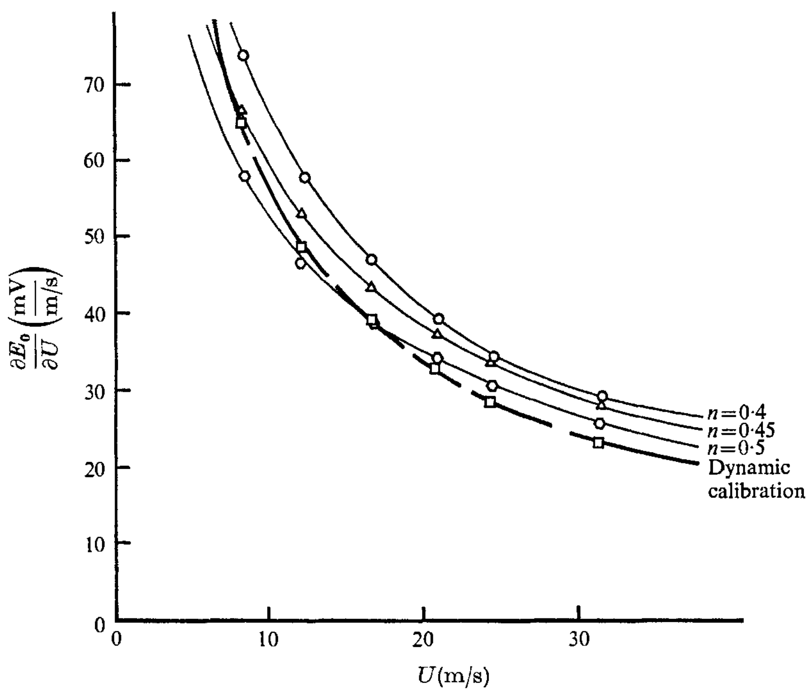

FIGURE 7. System sensitivity determined by static technique and dynamic technique.

The previously mentioned Karman vortex street tests provide a new flow situation for comparisons to be made between the static and dynamic calibration methods and also forms a direct verification of the system frequency response. If the dynamic calibration produces universal velocity perturbation profiles across the vortex streets this would give added confidence to the dynamic method. Hence it could be concluded that unaccounted phenomena such as wire vibration, 'strain gauging' and errors in processing the signals were absent during the dynamic calibration.

The results of the cylinder wake measurements of § 2 evaluated from the dynamic calibration are shown in figures 8 and 9 for the authors' and Disa systems respectively. These results are summarized in figure 10(*b*) and the results of the low-frequency response tests performed with the dynamic calibrator are shown in figure 10(*a*). Thus the correlation as suggested by (2) is satisfied when the dynamic calibration is used. These results were corrected for temperature changes after calibration. The main correction was due to the changes of the wire cold resistance, which amounts to approximately 0·5 % per °C. In these experiments the maximum ambient temperature variation was 2 °C. The results show that for bias velocities less than 10 m/s and a platinum hot wire 4 μ in

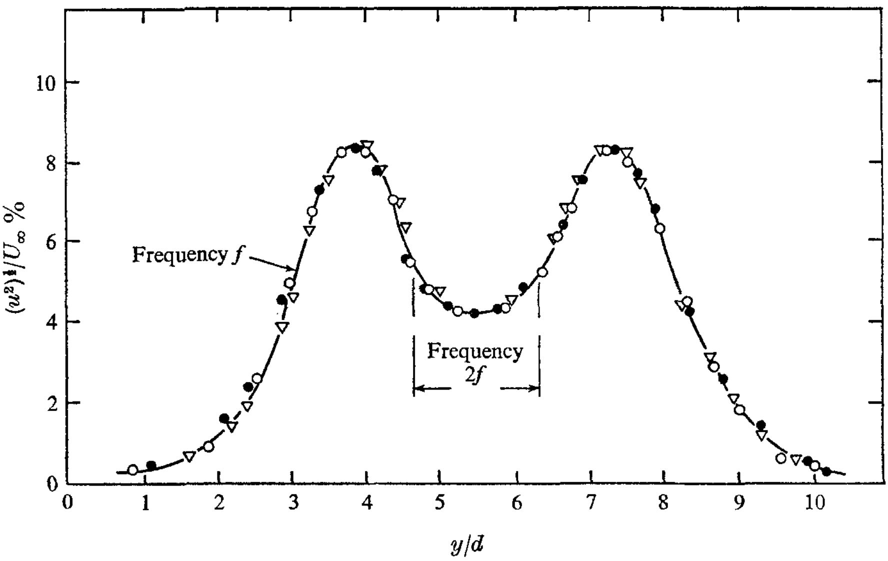

FIGURE 8. Velocity perturbation profiles as measured with authors' system. $R_n = 140$. (Dynamic calibration.) Frequency f (Hz): ▽, 500; ○, 1500; ●, 4000. Origin for y is arbitrary.

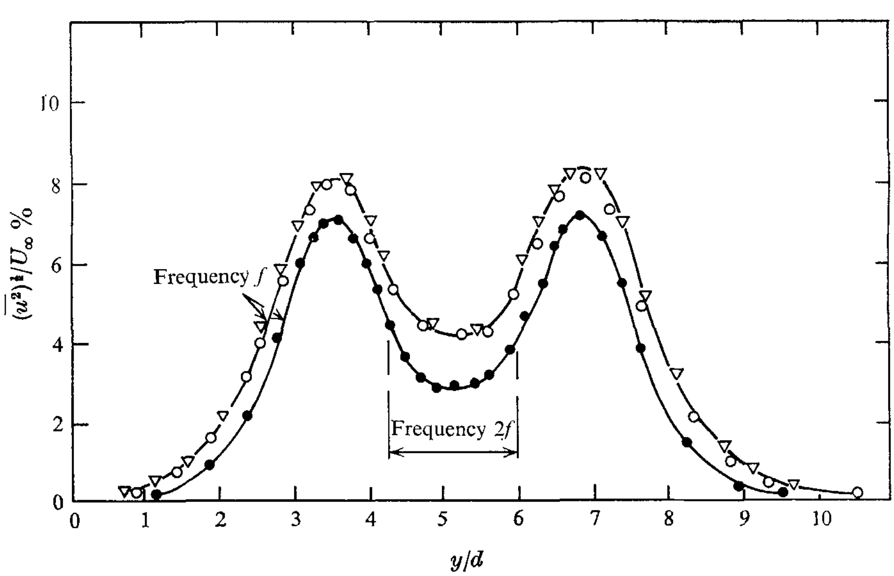

FIGURE 9. Velocity perturbation profiles as measured with Disa 55A01 system. $R_n = 140$. (Dynamic calibration.) Frequency f: ▽, 500; ○, 1500; ●, 4000. Origin for y is arbitrary.

diameter and 1·2 mm long the Disa type system has a 3 db frequency drop off at 6 kHz and the authors' system is flat beyond 10 kHz. The voltage stimulation tests indicate that the frequency response of the Disa is 5 kHz and 35 kHz for the authors' system. As a further check figure 11 shows the results for a given turbulence field determined from the dynamic calibration as a function of wire resistance ratio for the two systems investigated.

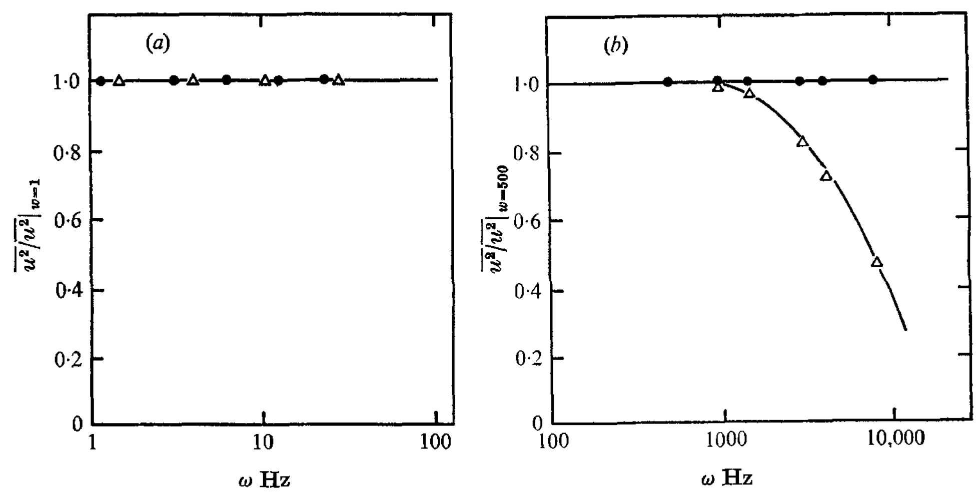

FIGURE 10. Normalized perturbation measurements from calibrator tests and vortex shedding tests. (*a*) Calibrator tests. (*b*) Vortex shedding tests. ●, authors' system; △, Disa 55A01 system.

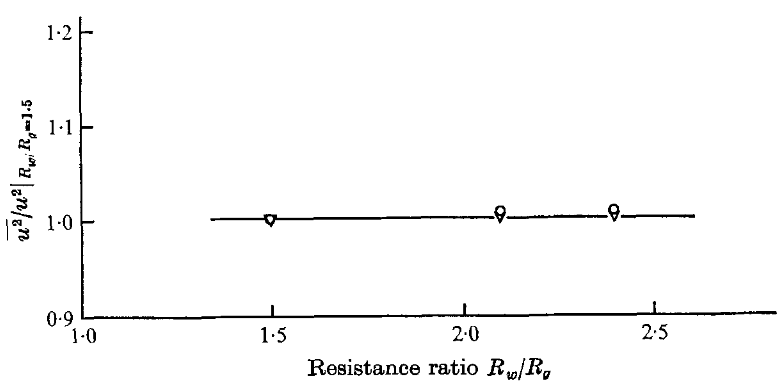

FIGURE 11. Velocity perturbation measurement as a function of resistance ratio using the dynamic calibration. ○, Disa 55A01; ▽, authors' system.

6. Non-linearity of hot-wire system

To determine the magnitude of the error introduced by large amplitude calibration signals the calibrator was used to generate velocity perturbation signals of up to $0{\cdot}4U$ r.m.s. Figure 12 shows the system sensitivity at one fixed bias velocity as a function of the velocity perturbation level. These results show that the error in the r.m.s. output voltage of the hot wire for a sinusoidal calibrator signal is less than 1 % provided the r.m.s. velocity perturbation is less than $0{\cdot}15U$. It must be noted that during this test there was a longitudinal velocity perturbation only, whereas during a boundary-layer or wake measurement there are other errors due to the transverse velocity perturbations. Tests including these effects are under examination. Any errors due to the transverse velocity perturbation in the cylinder wake measurement reported in § 2 will not affect the results since the error will be the same in all tests and thus the correlation suggested by equation (2) should still apply.

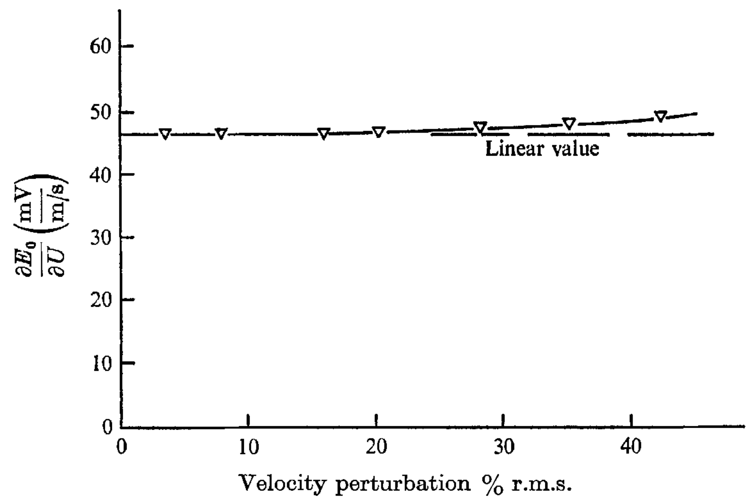

FIGURE 12. Non-linearity of system for longitudinal velocity perturbations.

7. Conclusions

Errors of more than 20 % may be encountered if the conventional static calibration technique is used. For a real hot wire this technique is theoretically correct provided the functional form of the heat transfer relation and its range of application is known with sufficient accuracy. However, in practice the static calibration method is subjective and the usual heat-transfer laws do not appear to be applicable. Another source of inaccuracy in this method is that it uses the slope of a curve fitted to scattered data. It appears that these difficulties may be overcome by using a dynamic small perturbation calibration technique similar to the one discussed in this paper. The vortex shedding test demonstrates the errors one obtains using the conventional calibration technique in contrast with the dynamic calibration. This test also determines the frequency response of the system.

The authors are indebted to the Australian Institute of Nuclear Science and Engineering for their financial support of this project.

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